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Robust Utility Design in Distributed Resource Allocation Problems with Defective Agents

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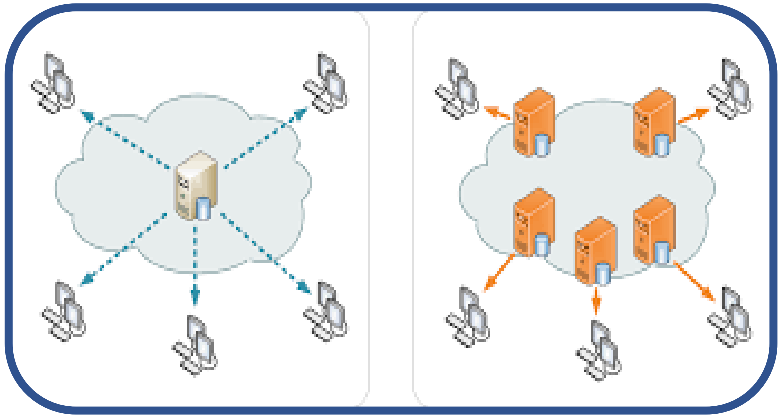
Multi-Agent Systems & Resource Allocation

Team & Task Assignment

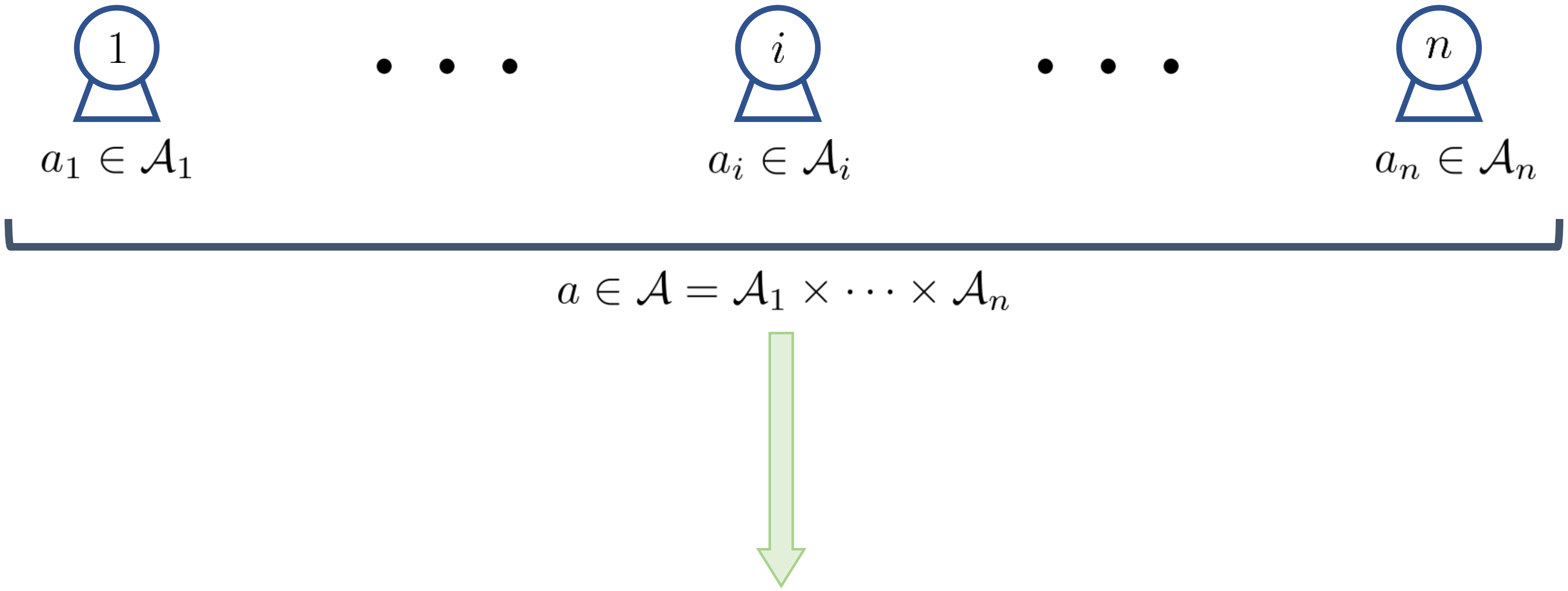
Fleets of Autonomous Robots



Content Distribution Networks

Multiple Funding Sources for Projects

Multi-Agent Systems



Agents

$1 \quad \cdots \quad i \quad \cdots \quad n$

Actions

$a_1 \in \mathcal{A}_1 \qquad a_i \in \mathcal{A}_i \qquad a_n \in \mathcal{A}_n$

Collective Action

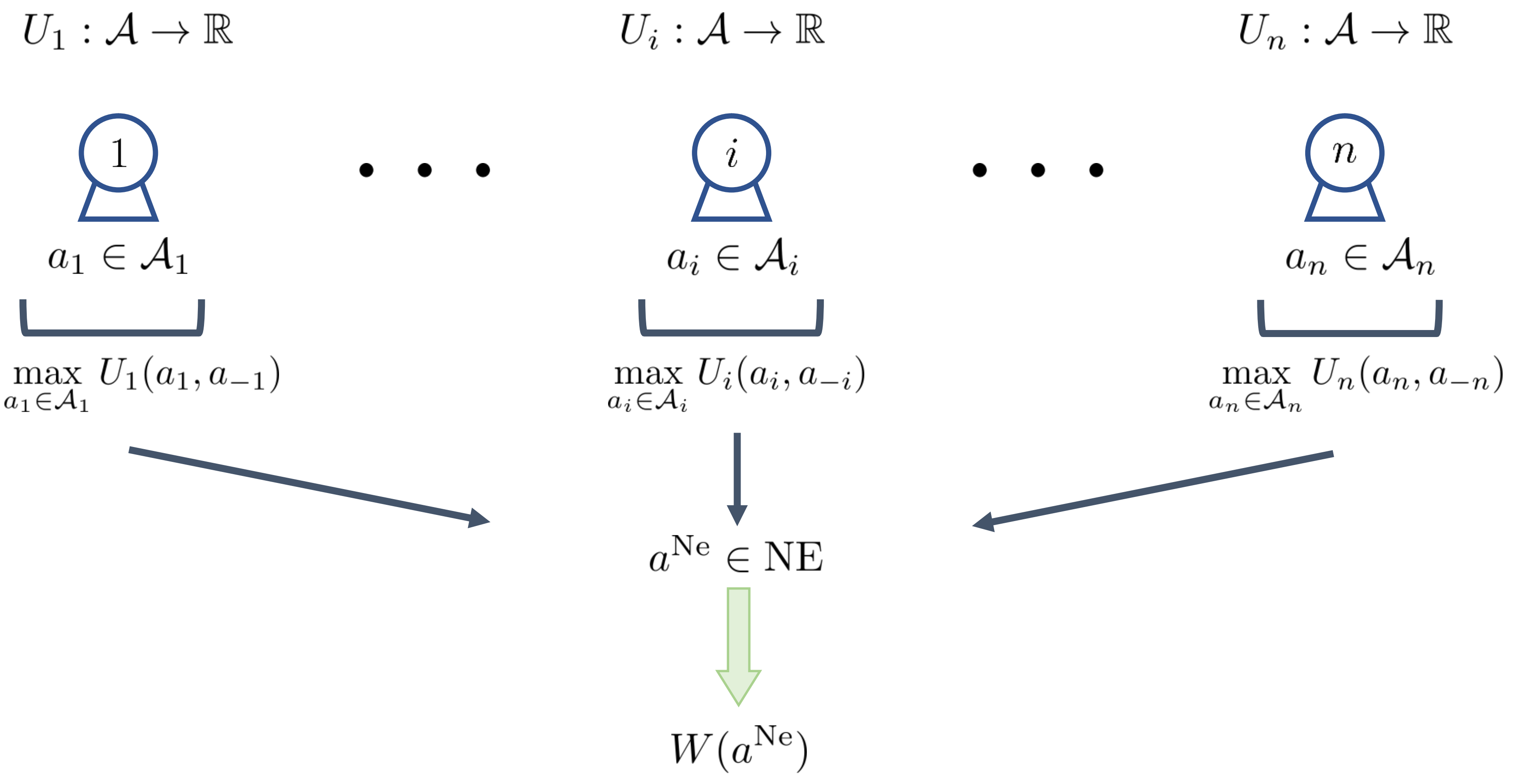
$$a \in \mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_n$$

System Welfare

$$a^{\text{opt}} \in \arg\max_{a \in \mathcal{A}} \; W(a)$$

Hard to find!

Multi-Agent Systems



Utility: $U_1 : \mathcal{A} \to \mathbb{R}$ … $U_i : \mathcal{A} \to \mathbb{R}$ … $U_n : \mathcal{A} \to \mathbb{R}$

Agents: 1 $\cdots$ i $\cdots$ n

Actions: $a_1 \in \mathcal{A}_1$ … $a_i \in \mathcal{A}_i$ … $a_n \in \mathcal{A}_n$

Individual Decision Making: $\max_{a_1 \in \mathcal{A}_1} U_1(a_1, a_{-1})$ … $\max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i})$ … $\max_{a_n \in \mathcal{A}_n} U_n(a_n, a_{-n})$

Equilibrium Action: $a^{\text{Ne}} \in \text{NE}$

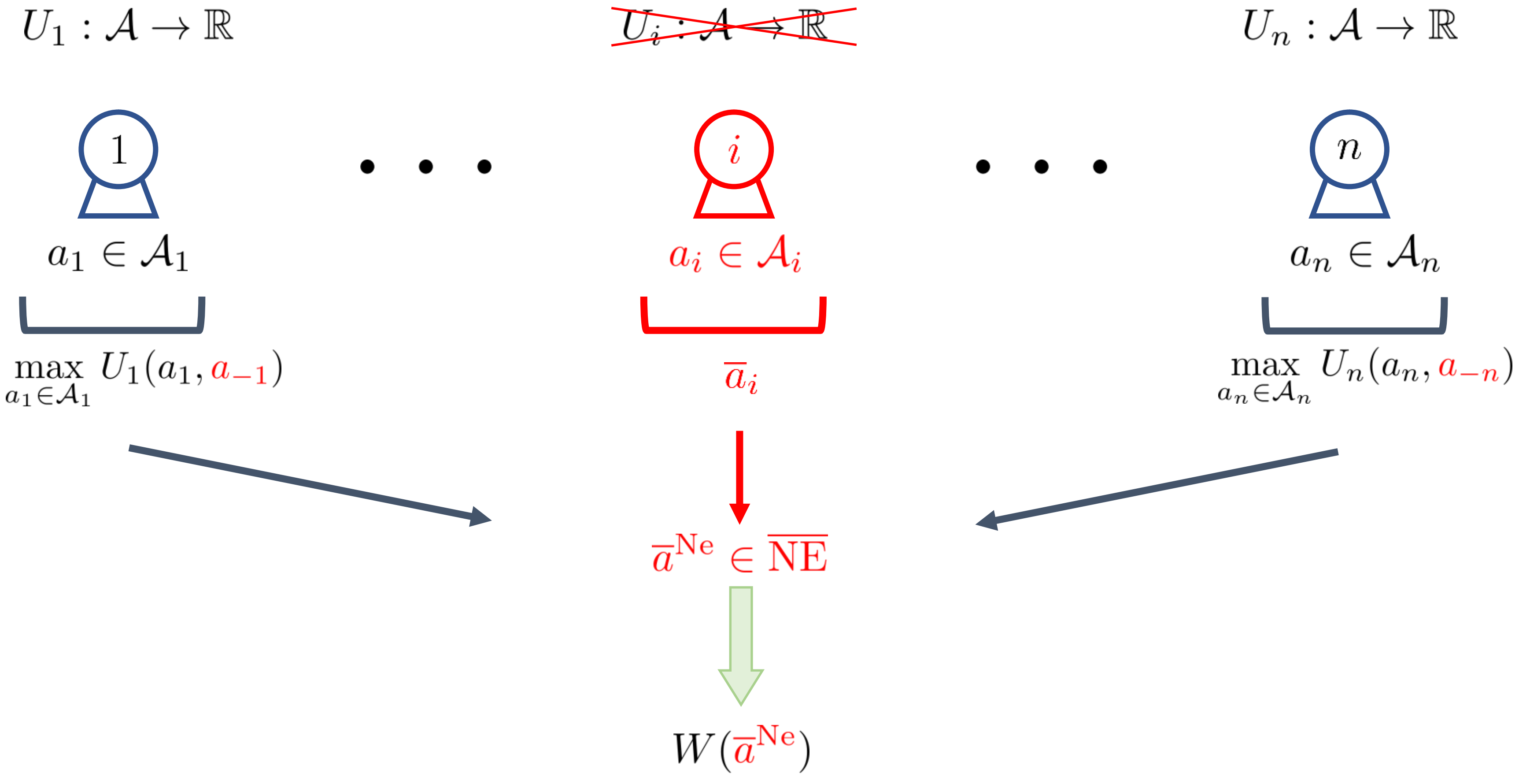
System Welfare: $W(a^{\text{Ne}})$

Efficiency:

$$\text{PoA} = \frac{\min_{a^{\text{Ne}} \in \text{NE}} W(a^{\text{Ne}})}{\max_{a^{\text{opt}} \in \mathcal{A}} W(a^{\text{opt}})} \leq 1$$

How well can distributed decision making approximate optimal?

Multi-Agent Systems – Defective Agents



Utility $U_1 : \mathcal{A} \to \mathbb{R}$ $\cancel{U_i : \mathcal{A} \to \mathbb{R}}$ $U_n : \mathcal{A} \to \mathbb{R}$

Agents 1 $\cdots$ i $\cdots$ n

Actions $a_1 \in \mathcal{A}_1$ $a_i \in \mathcal{A}_i$ $a_n \in \mathcal{A}_n$

Individual Decision Making $\max_{a_1 \in \mathcal{A}_1} U_1(a_1, a_{-1})$ $\overline{a}_i$ $\max_{a_n \in \mathcal{A}_n} U_n(a_n, a_{-n})$

Equilibrium Action $\overline{a}^{\mathrm{Ne}} \in \overline{\mathrm{NE}}$

System Welfare $W(\overline{a}^{\mathrm{Ne}})$

Efficiency

$$\mathrm{PoA} = \frac{\min_{\overline{a}^{\mathrm{Ne}} \in \overline{\mathrm{NE}}} W(\overline{a}^{\mathrm{Ne}})}{\max_{a^{\mathrm{opt}} \in \mathcal{A}} W(a^{\mathrm{opt}})} \leq 1$$

Multi-Agent Systems – Defective Agents

Utility $U_1 : \mathcal{A} \to \mathbb{R}$ $\xcancel{U_i : \mathcal{A} \to \mathbb{R}}$ $U_n : \mathcal{A} \to \mathbb{R}$

Agents 1 … i … n

How do ***defective agents*** affect system ***performance***?

Actions

Individual Decision Making $\max_{a_1 \in \mathcal{A}_1} U_1(a_1, a_{-1})$ $\overline{a}_i$ $\max_{a_n \in \mathcal{A}_n} U_n(a_n, a_{-n})$

How can we ***design*** utilities that are ***robust***?

Equilibrium Acti

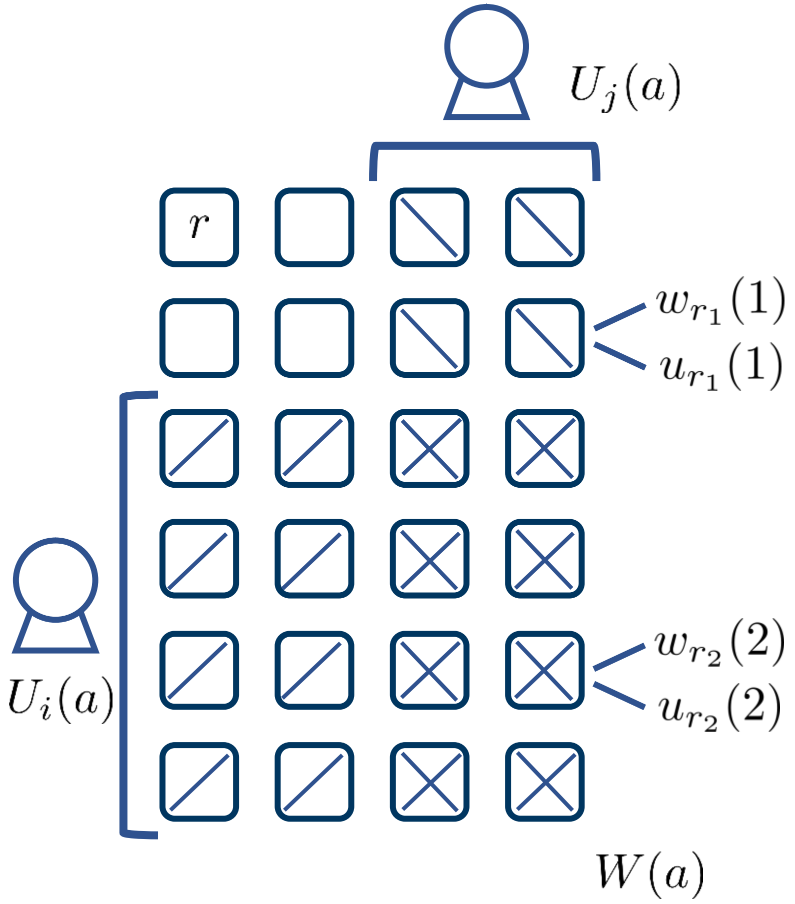
System Welfare $W(\overline{a}^{\mathrm{Ne}})$

Efficiency

$$\mathrm{PoA} = \frac{\min_{\overline{a}^{\mathrm{Ne}} \in \overline{\mathrm{NE}}} W(\overline{a}^{\mathrm{Ne}})}{\max_{a^{\mathrm{opt}} \in \mathcal{A}} W(a^{\mathrm{opt}})} \leq 1$$

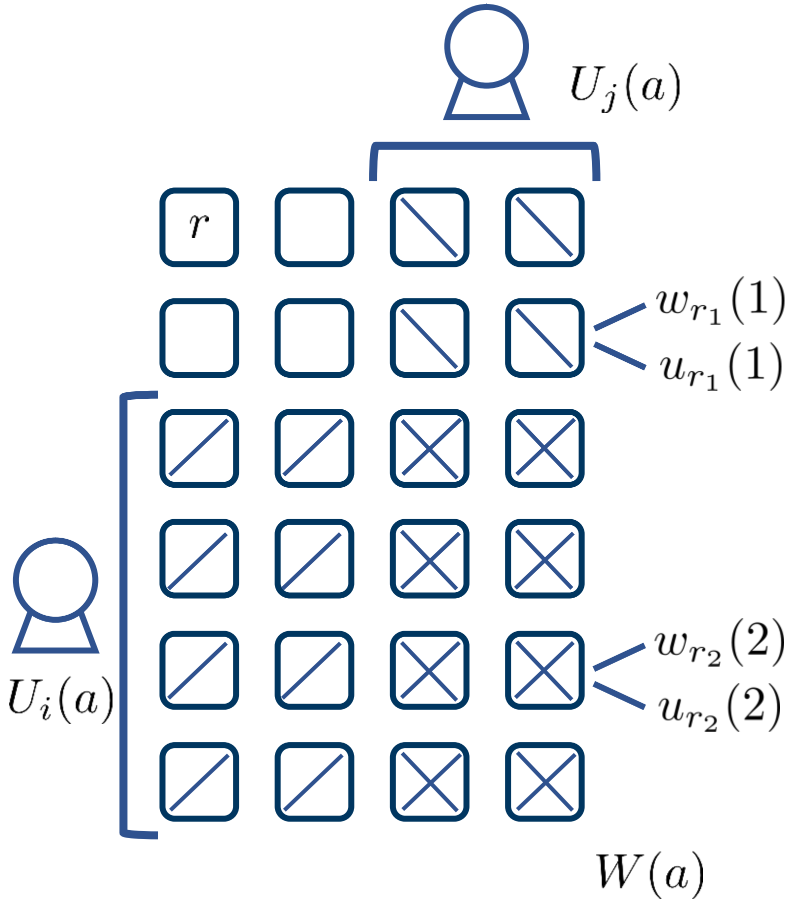
Resource Allocation Problems

Resources	$r \in \mathcal{R} = \{1, \ldots, R\}$
Agents	$i \in N = \{1, \ldots, n\}$
Actions	$a_i \in \mathcal{A}_i \subseteq 2^{\mathcal{R}}$
System Welfare	$W(a) = \sum_{r \in \mathcal{R}} \underbrace{w_r(\lvert a \rvert_r)}_{\text{Value of having } \lvert a \rvert_r \text{ agents share resource } r}$
Local Utility Rule	$U_i(a) = \sum_{r \in a_i} \underbrace{u_r(\lvert a \rvert_r)}_{\text{Utility of having } \lvert a \rvert_r \text{ agents share resource } r}$
Resource Allocation Problem	$G = (\mathcal{R}, N, \mathcal{A}, \{w_r, u_r\}_{r \in \mathcal{R}})$
Price of Anarchy	$\mathrm{PoA}(G) = \dfrac{\min_{a^{\mathrm{Ne}} \in \mathrm{NE}(G)} W(a^{\mathrm{Ne}})}{\max_{a^{\mathrm{opt}} \in \mathcal{A}} W(a^{\mathrm{opt}})} \leq 1$



Resource Allocation Problems

Resources	$r \in \mathcal{R} = \{1, \dots, R\}$
Agents	$i \in N = \{1, \dots, n\}$
Actions	$a_i \in \mathcal{A}_i \subseteq 2^{\mathcal{R}}$
System Welfare	$w_r \in \mathcal{W}$
Local Utility Rule	$u_r = \mathcal{U}(w_r)$
Class of Resource Allocation Problems	$\mathcal{G}_{\mathcal{W},\mathcal{U}} = \{G \mid w_r \in \mathcal{W}, u_r = \mathcal{U}(w_r)\ \forall r \in \mathcal{R}\}$
Price of Anarchy	$\mathrm{PoA}(\mathcal{G}_{\mathcal{W},\mathcal{U}}) = \inf_{G \in \mathcal{G}_{\mathcal{W},\mathcal{U}}} \mathrm{PoA}(G) \leq 1$



Goal: Design $\mathcal{U}$ to maximize efficiency guarantees

Stubborn Agents

Regular Agents $i \in N = \{1, \ldots, n\}$ $a \in \mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_n$

Stubborn Agents $j \in M = \{1, \ldots, m\}$ $d \subset 2^{\mathcal{R}}$

Fixed Action $d_j \in 2^{\mathcal{R}}$

System Welfare $W(a) = \sum_{r \in \mathcal{R}} w_r(\underbrace{|a|_r}_{\text{Only depends on regular agents}})$

Only depends on regular agents

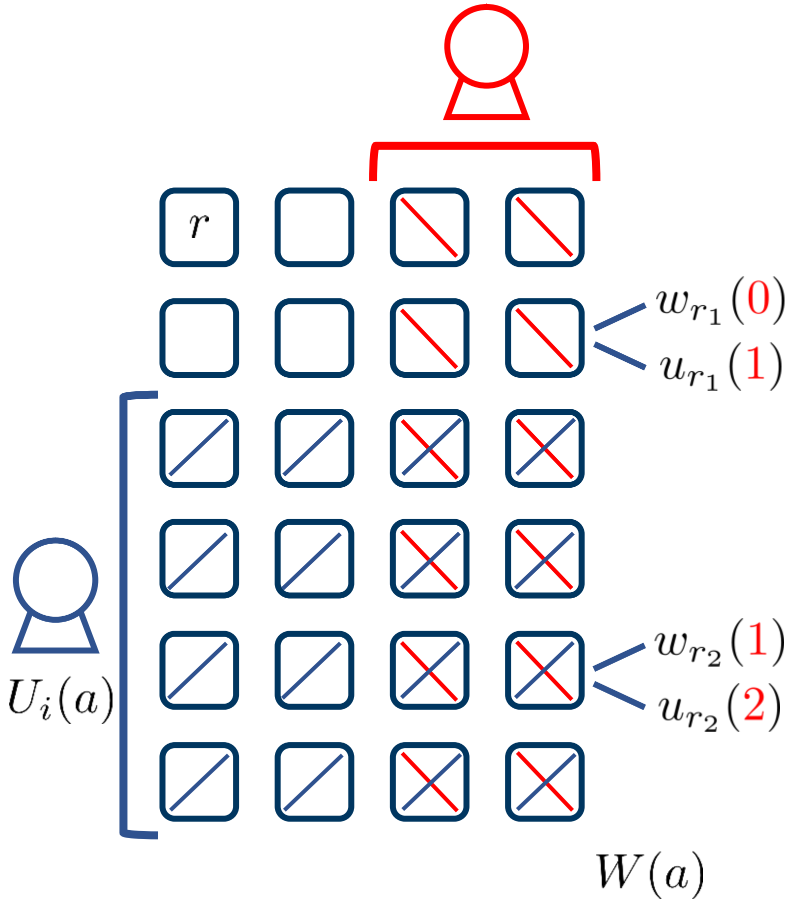
Local Utility Rule $U_i(a; d) = \sum_{r \in a_i} u_r(\underbrace{|a|_r + |d|_r}_{\text{Depends on regular and defective agents}})$

Depends on regular and defective agents

Perturbed Equilibria $\mathrm{NE}(G, d)$

Robust Efficiency Guarantee

$$\mathrm{PoA}(\mathcal{G}^m_{\mathcal{W},\mathcal{U}}) = \inf_{G \in \mathcal{G}_{\mathcal{W},\mathcal{U}}} \min_{d \subset 2^{\mathcal{R}} : |d| \leq m} \mathrm{PoA}(G, d) \leq 1$$



How should we design ***robust utility functions***?

Optimal Utility Rules

Proposition 1

Suppose there are at most m deffective agents. Let $\mathcal{W} = \{\sum_{t=1}^{T} \alpha_t w_t | \alpha_t \geq 0 \; \forall t \in [T]\}$ be a set of resource value functions. For each $t \in [T]$, let $(u_t^\star, \mu_t^\star)$ be the solution of the following linear program

$$\begin{aligned}(u_t^\star, \mu_t^\star) \in \underset{u \in \mathbb{R}^n,\ \mu \in \mathbb{R}}{\arg\max}\ & \mu \\ \text{s.t. } & w_t(z+y) - \mu w_t(x+y) \\ & \quad + xu(x+y+d) - zu(x+y+d+1) \leq 0 \\ & \forall (x,y,z) \in \mathcal{I}_n,\ d \in \{0,\ldots,m\},\end{aligned}$$

where $\mathcal{I}_n = \{(x,y,z) \in \mathbb{N}_{\geq 0}^3 \mid 1 \leq x+y+z \leq n\}$. The following statements hold true:

(i) There exists a linear local utility rule $\mathcal{U}^*$ that optimizes the price of anarchy. Furthermore, if $w = \sum_{t=1}^{T} \alpha_t w_t$, then $\mathcal{U}^*(w) = \sum_{t=1}^{T} \alpha_t u_t^*$.

(ii) The optimal price of anarchy satisfies $\mathrm{PoA}(\mathcal{G}_{\mathcal{W},\mathcal{U}^*}^m) = \max_{t \in [T]} 1/\mu_t^\star$.

Easy to find

Easy to construct

Robust Performance Guarantee

Covering Problems

Each resource has a value that is contributed to the welfare when covered by at least one agent

$$W(a) = \sum_{r \in \mathcal{R}} v_r \cdot \mathbb{1}[|a|_r > 0]$$

Theorem [Gairing 2009]

Optimal utility design

$$U_i(a) = \sum_{r \in a_r} v_r \cdot u^0(|a|_r)$$

where

$$u^0(x) = (x-1)! \frac{\frac{1}{(n-1)(n-1)!} - \sum_{i=x}^{n-1} \frac{1}{i!}}{\frac{1}{(n-1)(n-1)!} - \sum_{i=1}^{n-1} \frac{1}{i!}}$$

with

$$\mathrm{PoA}(\mathcal{G}_{\mathbb{1},u^0}) = 1 - \frac{1}{e} \approx 0.632$$

What do stubborn agents cause?

$$\mathrm{PoA}(\mathcal{G}_{\mathbb{1},u^0}^{m=2}) \approx 0.254$$

$$\mathrm{PoA}(\mathcal{G}_{\mathbb{1},u^0}^{m=5}) \approx 0.121$$

How much can we improve robustness?

&

What do we sacrifice in the nominal setting?

Trade-off in Covering

Theorem 2

In the class of covering games, if

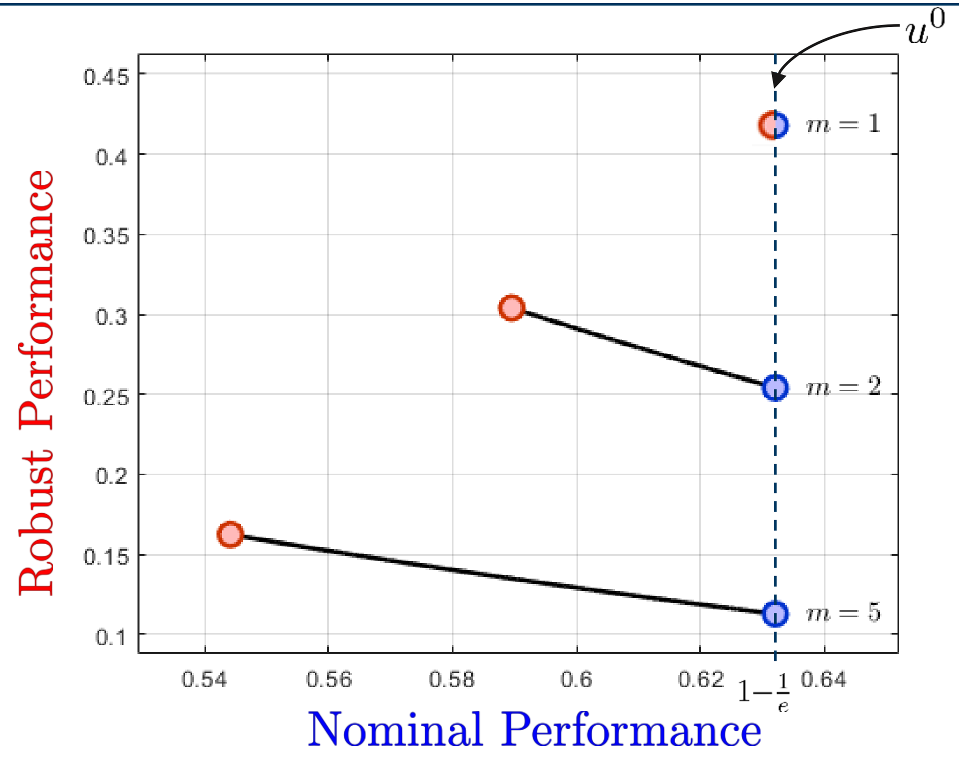
$$\mathrm{PoA}(\mathcal{G}^m_{\mathbb{1},u}) \geq \frac{\Gamma_m + \frac{e}{e-1}}{1 + t\Gamma_m},$$

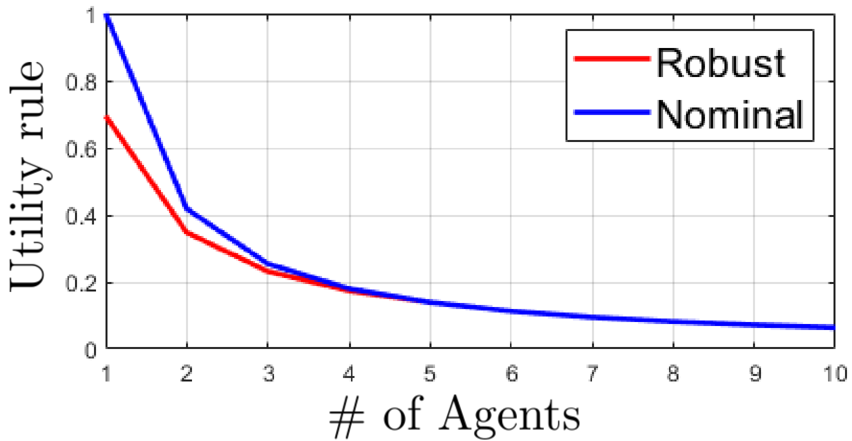
where $\Gamma_m = m!\frac{e - \sum_{i=0}^{m-1}\frac{1}{i!}}{e-1} - 1$ and $t \in [0,1]$, then

$$\mathrm{PoA}(\mathcal{G}^0_{\mathbb{1},u}) \leq \frac{(e-1)(1+t\Gamma_m)}{1+(e-1)(1+t\Gamma_m)}.$$

These price of anarchy guarantees can be realized by

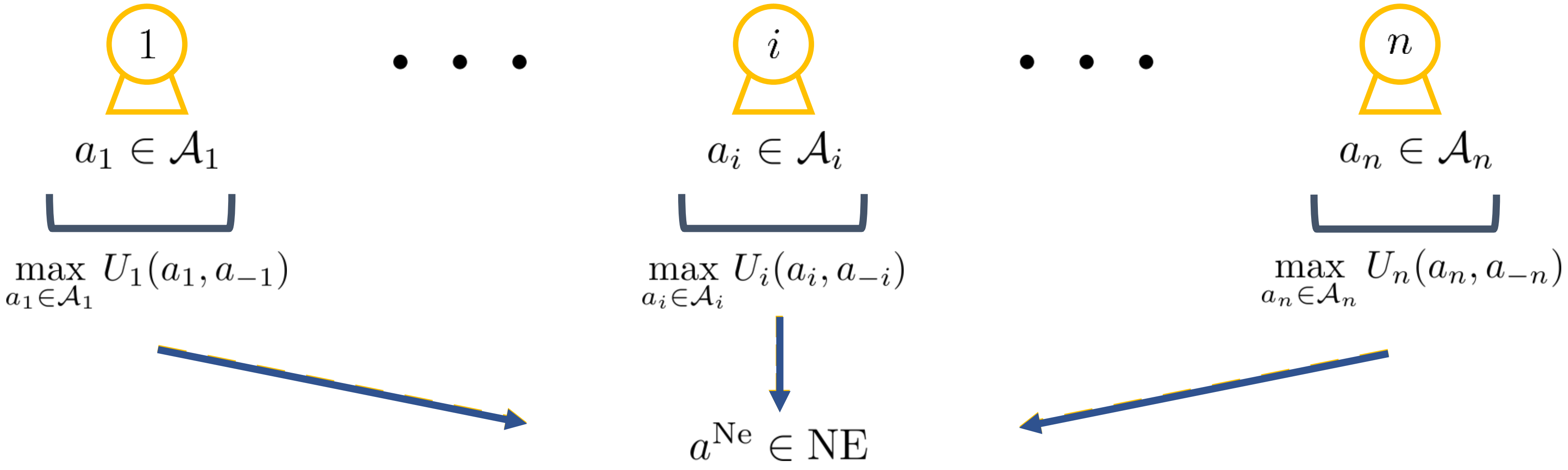
$$u^t(x) = u^0(x) - \max\left\{t\left(u^0(x) - \frac{m}{x}u^0(m)\right), 0\right\}.$$





Failure Prone Agents

Each agent ‘fails’ (does not contribute to the welfare) w.p. p



$a_1 \in \mathcal{A}_1$ $\quad$ $a_i \in \mathcal{A}_i$ $\quad$ $a_n \in \mathcal{A}_n$

$\max_{a_1 \in \mathcal{A}_1} U_1(a_1, a_{-1})$ $\quad$ $\max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i})$ $\quad$ $\max_{a_n \in \mathcal{A}_n} U_n(a_n, a_{-n})$

$a^{\mathrm{Ne}} \in \mathrm{NE}$

Failure Prone Agents

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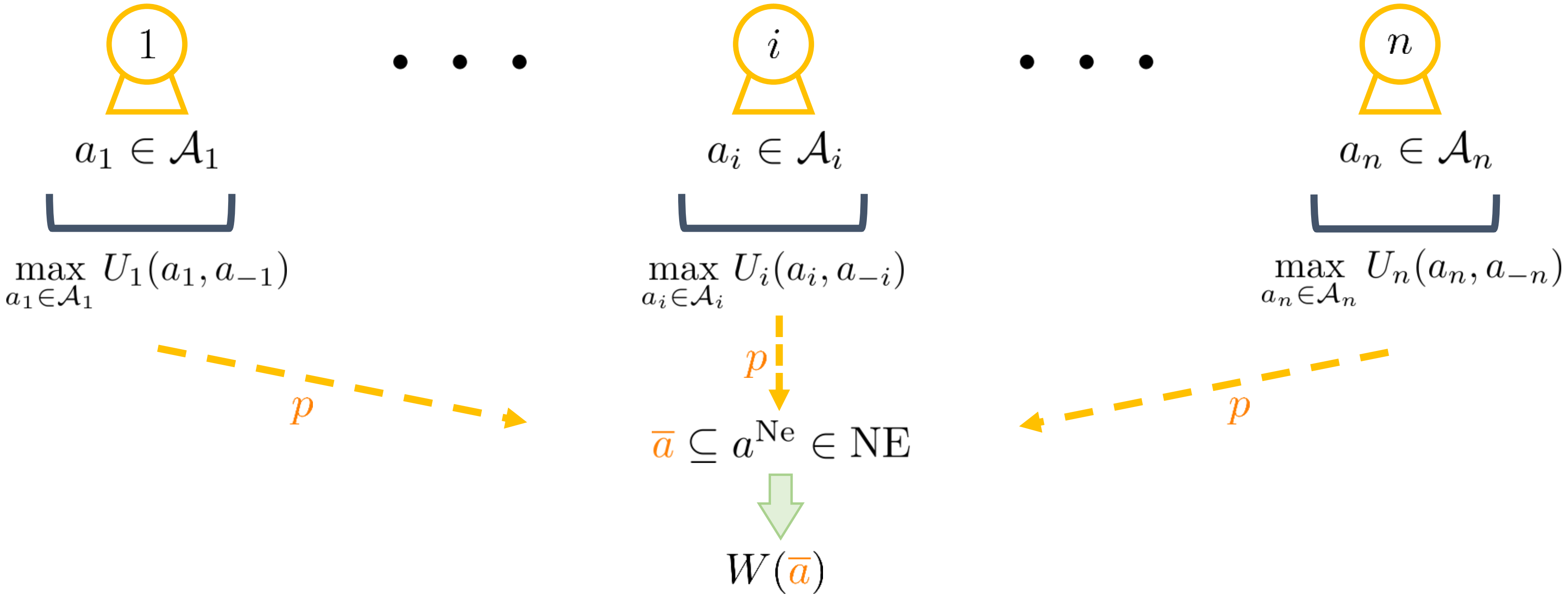


1 … i … n

$a_1 \in \mathcal{A}_1$ $\quad a_i \in \mathcal{A}_i$ $\quad a_n \in \mathcal{A}_n$

$\max_{a_1 \in \mathcal{A}_1} U_1(a_1, a_{-1})$ $\quad \max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i})$ $\quad \max_{a_n \in \mathcal{A}_n} U_n(a_n, a_{-n})$

p $\quad p$ $\quad p$

$\overline{a} \subseteq a^{\mathrm{Ne}} \in \mathrm{NE}$

$W(\overline{a})$

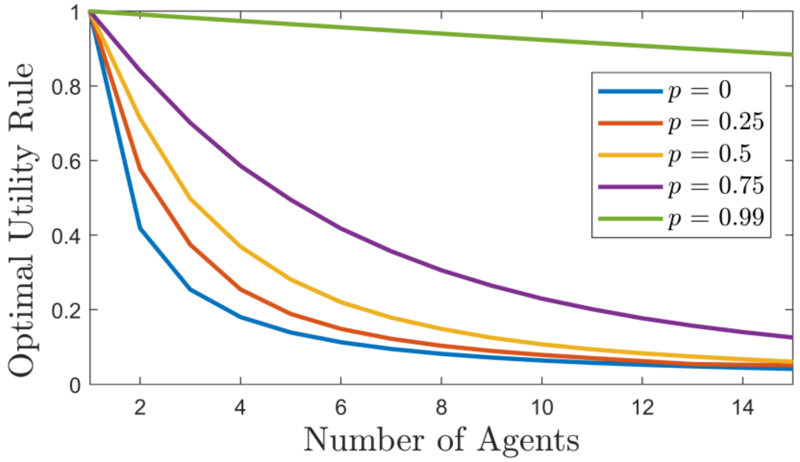
Corollary 1

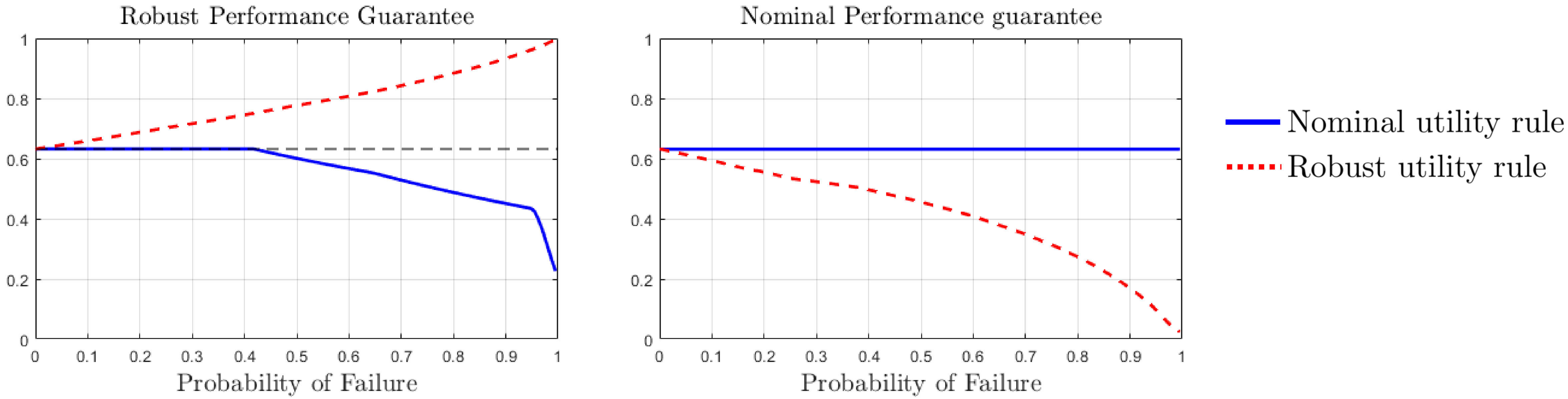
Suppose agents fail w.p. p. Let $\mathcal{W} = \{\sum_{t=1}^{T} \alpha_t w_t | \alpha_t \geq 0 \ \forall t \in [T]\}$ be a set of resource value functions. We can solve the LP from Proposition 1 with

$$\overline{w}^t(x) = \sum_{k=0}^{x} w^t(k) \binom{x}{k} (1-p)^k \ p^{x-k} \quad \forall \ t \in [T].$$

and get an optimal mechanism and price of anarchy guarantee.

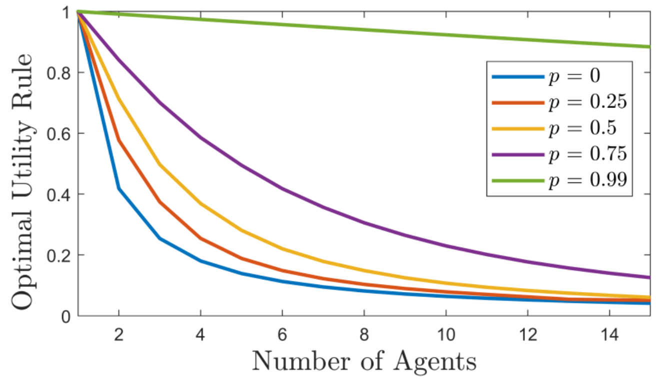
Failure Prone Agents in Covering Problems



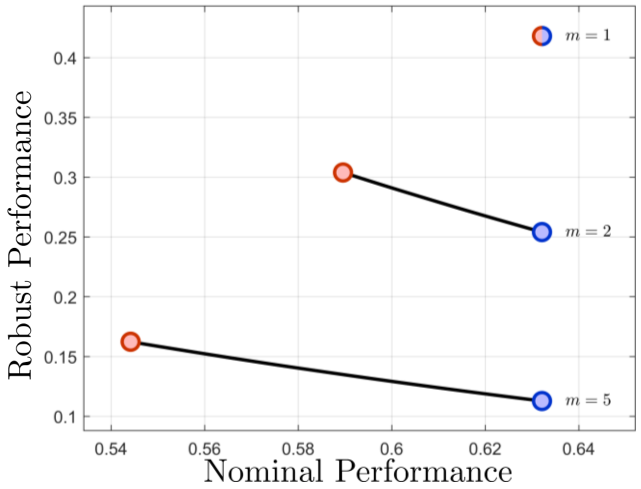


Conclusions

- For a more ***robust*** design, higher utility for more ***overlap***



- ***Trade-off*** between ***nominal*** and ***robust*** performance guarantees



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